Homework 1

SAAS DF

Spring 2025

Only problems marked with (*) are expected to be completed. However, the other ones could be fun, and a good primer for future classes or just for fun!

1 Muscle Memory

Here are some practice problems on differentiation and calculus. Don't worry if you find them difficult! We made them intentionally hard to push your limits. See the hints section at the end if you need some help, and we'll also host office hours if you need. Don't be afraid to slack the Education team!

Problem 1 (*). Find all partial derivatives of the following three functions:

- $f(x) = x^2 + 2xy + y^2$
- $g(x, y) = \sqrt{x^2 + y^2}$
- $h(x, y, z) = x^2 + 2y^2 + 3z^2 + 3xy + 4xz + 5yz$

Challenge (Optional, but healthy):

- $G(\mathbf{x}) = \sqrt{\sum_{i=1}^{N} x_i^2}$
- $H(\mathbf{x}) = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} x_i x_j$

Problem 2. Find prospective minima and maxima for each of the functions you computed the derivatives for above.

Problem 3 (Desmos Activity). Consider the function $y = x^4 + 2x^2 + 1$. Find the derivative of y with respect to x, $\frac{\partial y}{\partial x}$. Plot both y and $\frac{\partial y}{\partial x}$ in Desmos.
What do you notice about the derivative of the function around the minimum? Think about this quantiatively (what kinds of numbers are you seeing) and qualitatively (what kinds of shapes and vibes do things have)?

Challenge (Optional but healthy): Explain why even degree polynomials must have a minimum or maximum (no need to be rigorous, but if stuck use algebra to help: try thinking about the signs of the derivative as x goes to $-\infty$ and ∞, and if still stuck, see hint below).

2 Contour/Edge Detection (*)

Remarkably, partial derivatives and calculus have a direct application in contour and edge detection! We've used this method in DC projects in the past, so it'll be good to try! Please see `edge_detection.ipynb` for an introduction.

3 Sentiment Analysis

Often, we give an entire sentence just one sentiment score. But what if we wanted to track sentiment across a single body, along with other things like *momentum*? Please see `sentiments_over_time.ipynb` to try this!

4 Hints and Remarks

Remark (Muscle Memory, Notation). $\sum_{i=1}^{N} x_i^2$ just means $x_1^2 + x_2^2 + ... + x_N^2$. The bolded letter, $\mathbf{x}$, is just shorthand for $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}$. So $G(\mathbf{x})$ is the same as writing $G(x_1, x_2, ..., x_N)$.

Remark (Muscle Memory, Taking Partial Derivatives). Don't get hung up with the notation here! The hardest part of these problems is simply using your knowledge of the *algebraic properties of derivatives.*

- Derivatives split over sums! This means that:

$$\frac{\partial}{\partial x}(f+g) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial x}$$

  This is how we deal with it when we see a sum and a derivative.

- You can also take constants out!

$$\frac{\partial}{\partial x} cf = c\frac{\partial f}{\partial x}$$

On another note, have you seen these properties anywhere else?

Remark (Muscle Memory Challenge, Chain Rule). Chain rule can be confusing, especially when your function has many many layers. Here is a hint for the second problem, finding the partial derivatives of $g(\mathbf{x}) = \sqrt{\sum_{i=1}^{N} x_i^2}$. The trick is again to use *algebra*: define your own function, $g_1(\mathbf{x}) = \sum_{i=1}^{N} x_i^2$. Then $g(\mathbf{x}) = \sqrt{g_1(\mathbf{x})} = g_1(\mathbf{x})^{1/2}$. Chain rule then says:

$$\frac{\partial g}{\partial x_i} = \frac{1}{2} g_1(\mathbf{x})^{-1/2} \cdot \frac{\partial g_1}{\partial x_i}$$

Now you can proceed by evaluating each part separately, and then combining at the end later. As you practice, you will get more and more comfortable, so that you can do it all in your head with ease.

Remark (Muscle Memory Challenge, Ensuring Minima or Maxima). Why do all even degree polynomials have a minimum? There are many ways to see this, but one way comes from looking at the graph. Ask yourself these questions:

- Can the graph of an even degree polynomial go to negative infinity? [*The answer is no, but try to justify to yourself why using Desmos, and maybe even write it down in words (can help clear your head)!*]
- If the graph can't go to negative infinity, does that mean there has to be a minimum? [*You can answer this intuitively, to be totally precise this actually requires your function to be continuous.*]

This method of thinking about it doesn't necessarily need you to think about the derivative, but remember some of the reasoning for why we said x^2 should have a minimum.

Also, if you're curious about why this is a good result to know, this same reasoning can tell us that the **mean squared error (MSE)** has a minimum! Minimizing this function is very important to data science and machine learning. It would be pointless if it didn't have a minimum.